

## Universal Gravitation, Circular Motion, & Kepler's Laws

1. Read Chapter 7
2. Terms to know: universal gravitation, linear speed, rotational speed, centripetal acceleration, centripetal force, uniform circular motion, cycle, universal gravitational constant, gravitational field strength.
3. What is Newton's law of universal gravitation?  
**Every object in the universe is attracted to every other object in the universe.**
4. What happens to the gravitation force of attraction between two masses when:
  - a) the distance between the two masses is doubled?  
**Quartered**
  - b) the distance between the two masses is halved?  
**Quadrupled**
  - c) the mass of both objects is doubled?  
**Quadrupled**
  - d) the mass of one object is halved?  
**Halved**
5. Explain how what feels like a centrifugal force is really a centripetal force. Remember to use Newton's Laws appropriately. Provide 3 real world examples of this feeling.

You are continuing in a straight line motion, due to Newton's first law. There is an outside force acting on which is pushing or pulling you and forcing you to move in a circular path. It feels to you like you are being thrust to the outside of the circle, but really you are being directed inwards toward the center.

Turning a corner in a car, Turning a corner on a roller coaster, spinning around on a whirly amusement park ride, water skiing – jump the wake, merry-go-round

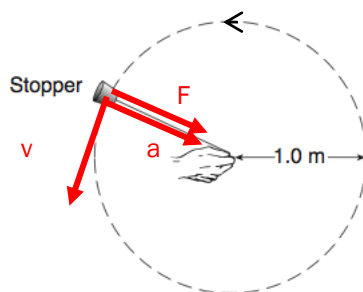
6. State Kepler's three laws of planetary motion (in your own words).
  - **#1: The path of the planets is elliptical in shape, with the center of the sun located at one foci.**
  - **#2: The straight line connected a planet and the sun will sweep out equal areas in equal amounts of time as it revolves around the sun.**
  - **#3: The ratio of the squares of the periods of any two planets is equal to the ratio of the cubes of their distances from the sun.**

**Directions:** Read each question carefully and record your answers in the space provided. Be sure to show all work! Answers should be in significant figures. You will be graded on proper use of the GUESS method. **These will be the same directions on the test. Practice the GUESS method now.**

7. A 3400 kg car is parked 23 meters away from a tractor that has a mass of  $5.6 \times 10^6$  kg. What is the force of gravitational attraction between the two vehicles?

$$F_G = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(5.6 \times 10^6 \text{ kg})(3400 \text{ kg})}{(23 \text{ m})^2} = 0.0024 \text{ N toward}$$

8. A student in lab twirls a rubber stopper on a 1.0 m string around in a circle overhead. It revolves 15 times in 3.74 seconds and has a mass of 7.2 kg.
- Draw and label the velocity, acceleration, and force vectors for the rubber stopper in the position shown on the diagram below.



(Not drawn to scale)

Velocity is tangential.  
Force and acceleration always point IN on a circle.

- Calculate its period.

$$T = \frac{t_{\text{total}}}{\# \text{ revolutions}} = \frac{3.74 \text{ s}}{15 \text{ rev}} = .249 \text{ s}$$

Sig figs match the total time

- Calculate its speed.

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.0 \text{ m})}{0.249 \text{ s}} = 25 \text{ m/s}$$

Distance is Circumference

- Calculate its centripetal acceleration.

$$a_c = \frac{v^2}{r} = \frac{(25 \frac{\text{m}}{\text{s}})^2}{1.0 \text{ m}} = 630 \frac{\text{m}}{\text{s}^2} \text{ in}$$

Acceleration always points IN on a circle

- Calculate the centripetal force acting on it.

$$F_c = ma_c = 7.2 \text{ kg}(630 \frac{\text{m}}{\text{s}^2}) = 4500 \text{ N in}$$

Force always points IN on a circle

9. A 2.00 kg mass is being swung in a circle of radius 2.00 m at a constant speed of 4.00 m/s.  
 a. What is the period of the object?

$$T = \frac{2\pi r}{v} = \frac{2\pi(2.00 \text{ m})}{(4.00 \frac{\text{m}}{\text{s}})} = 3.14 \text{ s}$$

- b. What is the magnitude of the tension that is needed to keep the object moving in a circle?

$$F_c = ma_c = \frac{mv^2}{r} = \frac{2.00 \text{ kg} (4.00 \frac{\text{m}}{\text{s}})^2}{2.00 \text{ m}} = 16.0 \text{ N}$$

“Magnitude of...” means no direction

10. A force of 3.4 newtons is needed to turn the handle of a 0.15 meter long wrench. Calculate the magnitude of the torque applied.

$$\tau = Fr = (3.4 \text{ N})(0.15 \text{ m}) = 0.51 \text{ Nm}$$

Honors Equation – need to memorize

11. Jupiter, which is located an average distance of  $7.78 \times 10^{11}$  meters from the sun has a period of 11.9 years. If Pluto is located  $5.91 \times 10^{12}$  meters from the sun, determine the time it would take for Pluto to make one complete revolution around the sun.

$$T_P = \sqrt{\frac{T_J^2 \cdot R_P^3}{R_J^3}} = \sqrt{\frac{(11.9 \text{ yr})^2 \cdot (5.91 \times 10^{12} \text{ m})^3}{(7.78 \times 10^{11} \text{ m})^3}} = 249 \text{ years}$$

Honors Equation – need to memorize

12. As the mass of a body increases, its gravitational force of attraction on the Earth

(A) Increases (B) Decreases (C) Remains the Same

$$F_g = mg$$

13. The magnitude of the gravitational force of attraction between the Earth and the Moon is approximately

(A)  $6.7 \times 10^{41}$  N (B)  $1.9 \times 10^{20}$  N (C)  $7.8 \times 10^{28}$  N (D)  $6.0 \times 10^{24}$  N

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2})(7.35 \times 10^{22} \text{ kg})(5.98 \times 10^{24} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2}$$

14. Gravitational force  $F$  exists between point objects A and B separated by a distance  $R$ . If the mass of A is doubled and the distance  $R$  is tripled, what is the new gravitational force?

(A)  $\frac{9}{2}F$  (B)  $\frac{3}{2}F$  (C)  $\frac{2}{3}F$  (D)  $\frac{2}{9}F$

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{1 \cdot 2 \cdot 1}{3^2} = \frac{2}{9}$$

15. As a cart travels around a horizontal circular track, the cart *must* undergo a change in

(A) weight (B) velocity (C) inertia (D) speed

Direction changes

16. An amusement park ride moves a rider at a constant speed of 14 meters per second in a horizontal circular path of radius 10. meters. What is the rider's centripetal acceleration in terms of g, the acceleration due to gravity?

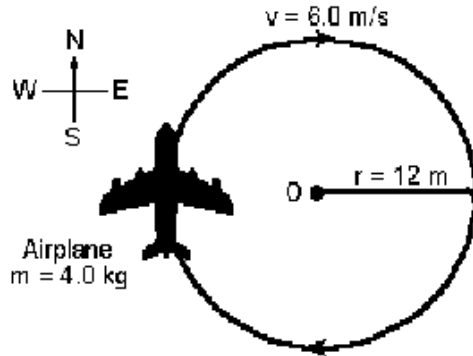
- (A) 0g      (B) 1g      (C) 2g      (D) 3g

$$a_c = \frac{v^2}{r} = \frac{(14 \frac{m}{s})^2}{10.m} = 19.6 \frac{m}{s^2} \text{ in}$$

$$\frac{19.6}{9.81} = 2g \quad (\text{How many "g=9.81" is the acceleration?})$$

Questions 17 through 19 refer to the following:

A 4.0 kilogram model airplane travels in a horizontal circular path of radius 12 meters at a constant speed of 6.0 meters per second.



17. At the position shown, what is the direction of the net force acting on the airplane?

- (A) east      (B) west      (C) north      (D) south

Force always points  
IN on a circle

18. What is the magnitude of the centripetal acceleration of the airplane?

- (A) 0.50 m/s<sup>2</sup>      (B) 2.0 m/s<sup>2</sup>      (C) 3.0 m/s<sup>2</sup>      (D) 12 m/s<sup>2</sup>

$$a_c = \frac{v^2}{r} = \frac{(6.0 \frac{m}{s})^2}{12m}$$

19. If the speed of the airplane is doubled and the radius of the path remains unchanged, the magnitude of the centripetal force acting on the airplane will be

- (A) half as much      (C) twice as much  
(B) one-fourth as much      (D) four times as much

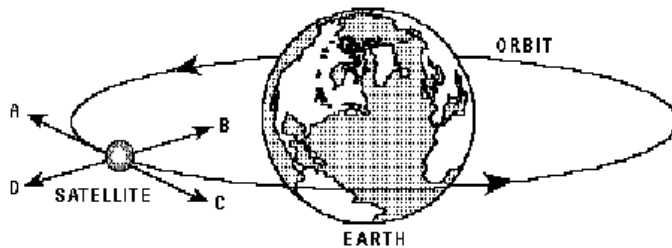
$$F_c = \frac{mv^2}{r} = \frac{1 \cdot 2^2}{1}$$

20. A motorcycle travels around a flat circular track. If the speed of the motorcycle increased, the force required to keep it in the same circular path

- (A) Increases      (B) Decreases      (C) Remains the Same

$$F_c = \frac{mv^2}{r}$$

21. A satellite is moving at constant speed in a circular orbit around the Earth, as shown in the diagram below.



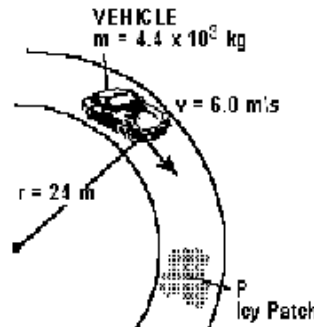
Force always points  
IN on a circle

The net force acting on the satellite is directed toward point

- (A) A      (B) B      (C) C      (D) D

Questions 22 and 23 refer to the following:

A vehicle travels at a constant speed of 6.0 meters per second around a horizontal circular curve with a radius of 24 meters. The mass of the vehicle is  $4.4 \times 10^3$  kilograms. An icy patch is located at point P on the curve.

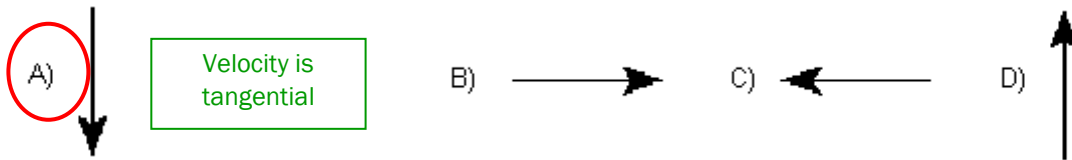


$$F_c = \frac{mv^2}{r} = \frac{(4.4 \times 10^3 \text{ kg})(6.0 \frac{\text{m}}{\text{s}})^2}{24 \text{ m}}$$

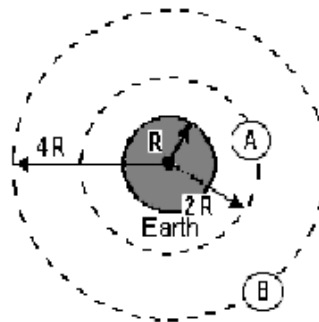
22. What is the magnitude of the frictional force that keeps the vehicle on its circular path?

- (A)  $1.1 \times 10^3 \text{ N}$       (B)  $6.6 \times 10^3 \text{ N}$       (C)  $4.3 \times 10^4$       (D)  $6.5 \times 10^4 \text{ N}$

23. On the icy patch of pavement, the frictional force on the vehicle is zero. What arrow best represents the direction of the vehicle's velocity when it reaches icy patch P?



24. Satellites A and B are orbiting the Earth in circular orbits as shown below. The mass of satellite A is twice as great as the mass of satellite B. Earth has a radius R.



Kepler's Third Law - objects orbiting further away take longer

Compared to the orbital period of satellite A, the orbital period of satellite B is

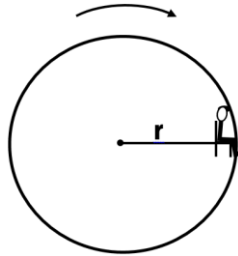
- (A) shorter      (B) longer      (C) the same

- Answers:  
 7. 0.0024 N toward  
 8. a. See sheet in classroom  
    b. 0.249 s  
    c. 25 m/s  
    d. 630 m/s<sup>2</sup> in  
    e. 4500 N in  
 9. a. 3.14 s  
    b. 16.0 N  
 10. 0.51 Nm  
 11. 249 years  
 12. A  
 13. B  
 14. D  
 15. B  
 16. C  
 17. A  
 18. C  
 19. D  
 20. A  
 21. B  
 22. B  
 23. A  
 24. B



Questions 10 – 12

A 30 kg child sits on the edge of a carnival ride at a radius of 2 m. The ride makes 2 revolutions in 4 s.



10. The period of the revolution is

- A)  $\frac{1}{2}$  rev/s
- B)  $\frac{1}{2}$  s
- C) 2 rev/s
- D) 2 s
- E) 4 s

11. The speed of the child is most nearly

- A) 4 m/s
- B) 6 m/s
- C) 24 m/s
- D) 120 m/s
- E) 360 m/s

12. The centripetal force acting on the child is most nearly

- A) 30 N
- B) 180 N
- C) 300 N
- D) 540 N
- E) 4,320 N

13. Two planets, A and B, orbit a star. Planet A moves in an elliptical orbit whose semimajor axis has length  $a$ . Planet B moves in an elliptical orbit whose semimajor axis has a length of  $9a$ . If planet A orbits with a period  $T$ , what is the period of planet B's orbit?

- A) 729 T
- B) 27 T
- C) 3 T
- D)  $T/3$
- E)  $T/27$

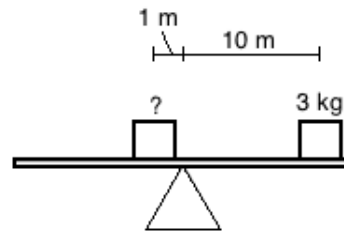
14. Torque

- A) is the vector product of force and lever arm displacement.
- B) is a scalar and has no direction associated with it.
- C) is always equal to force.
- D) is always greater for shorter lever arms.
- E) must always equal zero.

15. Which of the following are means of maximizing the torque of a force applied to a rotating object?

- I. Maximize the magnitude of the applied force.
- II. Apply the force as close as possible to the axis of rotation.
- III. Apply the force perpendicular to the displacement vector between the axis of rotation and the point of applied force

- A) I only
- B) II only
- C) I and II only
- D) I and III only
- E) I, II, and III



16. Two objects rest on a seesaw. The first object has a mass of 3 kg and rests 10 m from the pivot. The other rests 1 m from the pivot. What is the mass of the second object if the seesaw is in equilibrium?

- A) 0.3 kg
- B) 3 kg
- C) 10 kg
- D) 30 kg
- E) 50 kg