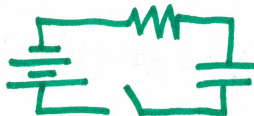


AP Review # 18

4. (10 points, suggested time 20 minutes)

Some students are investigating the behavior of a circuit with four components in series: a resistor of resistance  $R$ , a capacitor of capacitance  $C$ , a battery with potential difference  $\mathcal{E}$ , and a switch. Initially, the capacitor is uncharged and the switch is open.



• initially uncharged  $C$   
 • switch open

- (a)
- i. Determine the current in the resistor and the potential difference across the capacitor immediately after the switch is closed.
  - ii. Determine the current in the resistor and the potential difference across the capacitor a long time after the switch is closed.
- (b) The switch is opened, the capacitor is discharged, and a second, identical capacitor is added to the circuit in series with the other components. The switch is then closed again.
- i. A long time after the switch is closed, the energy stored in the single capacitor in the original circuit is  $U_1$ , and the total energy stored in the two capacitors in the new circuit is  $U_2$ . Calculate the ratio  $U_1/U_2$ .
  - ii. The two capacitors in series are to be replaced with a single capacitor that will have the same energy  $U_2$ . Indicate a plate area and a distance between the plates for the new capacitor, compared with one of the original capacitors, that will accomplish this. Support your reasoning using appropriate physics principles and/or mathematical models.

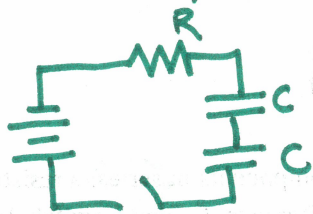
The students are then asked to design two circuits each containing a switch, a battery with a small internal resistance, a lightbulb, and a capacitor. In arrangement 1, the bulb should gradually light up after the switch is closed, becoming brightest after the switch has been closed a long time. In arrangement 2, the bulb should be brightest when the switch is first closed, getting dimmer with time, and going out completely when the switch has been closed for a long time.

- (c) Using standard symbols, draw two circuit diagrams, one showing a possible circuit for arrangement 1 and the other showing a possible circuit for arrangement 2. Justify your circuit diagrams with a paragraph-length explanation referring to the properties of lightbulbs and capacitors in circuits and the conservation of energy and/or the conservation of charge.

2a) i)  $I_R = \frac{V_T}{R_T} = \frac{\mathcal{E}}{R}$  (1)  $V_C = 0V$  immediately after closing

ii  $I_R = 0A$  (1)  $V_C = \mathcal{E}$  long time later

b) open switch, discharge C, add 2nd C in series



2 i) long time  $U_1 = 1st\ circuit$   $U_2 = 2nd\ circuit$   
ratio  $U_1:U_2$

$$U_C = \frac{1}{2} Q \Delta V = \frac{1}{2} C \Delta V^2$$

single:

$$C_T = C$$

$$\Delta V_T = \mathcal{E}$$

$$U_1 = \frac{1}{2} C \mathcal{E}^2$$

double:

$$C_T = \frac{1}{2} C$$

$$\Delta V_T = \mathcal{E}$$

$$U_2 = \frac{1}{2} \left( \frac{1}{2} C \right) (\mathcal{E})^2$$

$$U_1:U_2 \quad \boxed{2:1} \quad (i) \text{ correct ratios}$$

(i) eqn shows

$$V_{each} = \frac{1}{2}$$

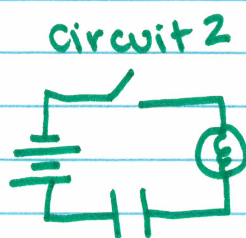
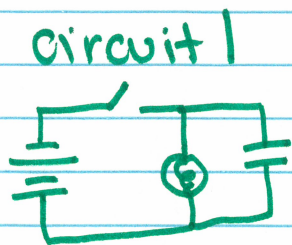
$$\text{or } C_T = \frac{1}{2}$$

1 ii  $C = \frac{k \epsilon_0 A}{d}$   $C_T = \frac{1}{2} C$

To achieve a  $C_T$  of  $\frac{1}{2} C_{single}$  you would either need to halve the area of the plates or double the distance.

(i)

5 c)



(i) correct diagrams

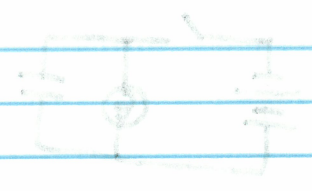
(i) bright when  $I = \text{max}$   
(i) use I or V  
(i) correct II  
(i) paragraph

In circuit 1, the capacitor is in parallel with the bulb. When a capacitor is <sup>first</sup> connected it acts like a bare wire, allowing all of the current to flow through it, bypassing the bulb. With no current, the bulb does not light. As the capacitor becomes charged, it begins to resist the flow of charges, allowing more charges to move through the other path (the bulb) so that path will gradually get brighter. Eventually the capacitor reaches steady state. No charges will flow through that path, so all of the current goes to the bulb.

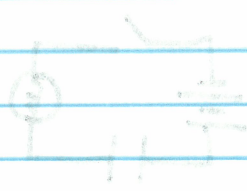
In circuit 2 the capacitor is in series with the bulb. There is only one path for current to take. When first connected the capacitor acts like a bare wire, allowing charge to flow through the whole circuit. The bulb gets the full share of the current and potential difference.

As the capacitor charges it takes a larger share of the voltage, decreasing the amount the bulb gets. Also the capacitor begins to prevent the flow of charges. This limits the current flowing through the bulb. With less current + potential difference, the bulb gets dimmer.

2c)



circuit 2



2c) circuit diagrams

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In circuit 1, the capacitor is in parallel with the bulb. When a capacitor is connected in parallel, it acts like a short circuit, allowing all of the current to flow through it, bypassing the bulb. Thus, no current flows through the bulb, and it does not light. As the capacitor becomes charged, it begins to resist the flow of current, allowing more current to flow through the bulb. The bulb will glow more brightly as the capacitor charges. Eventually, the capacitor will be fully charged and will resist all current flow, so all of the current goes through the bulb.

In circuit 2, the capacitor is in series with the bulb. This is only an open circuit for a short time. When first connected, the capacitor acts like a short circuit, allowing current to flow through the whole circuit. The bulb will glow. As the capacitor and battery begin to charge, the voltage across the capacitor increases and takes a larger share of the voltage, decreasing the amount the bulb receives. After the capacitor begins to charge, the current will stop. The bulb will stop glowing through the bulb. When the current stops, the bulb will stop glowing.